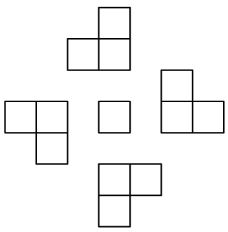
Advanced AIME Problems Part 2

SEM AMC Club - Compiled by Arjun Vikram, 2019

1. Find the number of five-digit positive integers, n, that satisfy the following conditions:
   - The number n is divisible by 5
   - The first and last digits of n are equal
   - The sum of the digits of n is divisible by 5

2. Let $ABCD$ be a square, and let E and F be points on $\overline{AB}$ and $\overline{BC}$, respectively. The line through E parallel to $\overline{BC}$ and the line through F parallel to $\overline{AB}$ divide $ABCD$ into two squares and two nonsquare rectangles. The sum of the areas of the two squares is $\frac{9}{10}$ of the area of square $ABCD$. Find $\frac{AE}{EB} + \frac{EB}{AE}$. *(Source: 2013 AIME 1 #3)*

3. In the array of 13 squares shown below, 8 squares are colored red, and the remaining 5 squares are colored blue. If one of all possible such colorings is chosen at random, the probability that the chosen colored array appears the same when rotated 90° around the central square is $\frac{1}{n}$, where n is a positive integer. Find n. *(Source: 2013 AIME 1 #4)*



4. The real root of the equation $8x^3 - 3x^2 - 3x - 1 = 0$ can be written in the form $\frac{\sqrt[3]{a} + \sqrt[3]{b} + 1}{c}$, where a, b, and c are positive integers. Find $a + b + c$. *(Source: 2013 AIME 1 #5)*

5. Let $\triangle\ PQR$ be a triangle with $\angle P = 75^\circ$ and $\angle Q = 60^\circ$. A regular hexagon $ABCDEF$ with side length 1 is drawn inside $\triangle PQR$ so that side $\overline{AB}$ lies on $\overline{PQ}$, side $\overline{CD}$ lies on $\overline{QR}$, and one of the remaining vertices lies on $\overline{RP}$. There are positive integers $a, b, c,$ and d such that the area of $\triangle PQR$ can be expressed in the form $\frac{a + b\sqrt{c}}{d}$, where a and d are relatively prime, and c is not divisible by the square of any prime. Find $a + b + c + d$. *(Source: 2013 AIME 1 #12)*

Answers: 1. 200 2. 018 3. 429 4. 098 5. 021