# Week 2: Advanced problems

Solutions will not be provided, however, you can find the solutions to all of the questions online at AoPS.com.

1. Let $S$ be the number of ordered pairs of integers $(a, b)$ with $1 \leq a \leq 100$ and $b \geq 0$ such that the polynomial $x^2 + ax + b$ can be factored into the product of two (not necessarily distinct) linear factors with integer coefficients. Find the remainder when $S$ is divided by $1000$. *(Source: 2018 AIME I Problem 1)$*.

2. Let $a_0 = 2,\ a_1 = 5,\ a_2 = 8$ and for $n > 2$ define $a_n$ recursively to be the remainder when $4(a_{n-1} + a_{n-2} + a_{n-3})$ is divided by $11$. Find $a_{2018} \cdot a_{2020} \cdot a_{@2022}$. *(Source: 2018 AIME II Problem 2)

3. Find a polynomial $p(x)$ of minimum degree such that $p(\sqrt{7} - \sqrt{3}) = 0$.

4. A strictly increasing sequence of positive integers $a_1, a_2, a_3, \cdots$, has the property that for every positive integer$k$, the subsequence $a_{2k-1}, a_{2k}, a_{2k+1}$ is geometric and the subsequence $a_{2k}, a_{2k+1}, a_{2k+2}$ is arithmetic. That is, the sequence $a_1, a_2, a_3$ is geometric, the sequence $a_2, a_3, a_4$ is arithmetic, the sequence $a_3, a_4, a_5$ is geometric, the sequence $a_4, a_5, a_6$ is arithmetic, and so on so forth. Suppose that $a_{13} = 2016$. Find $a_1$. *(Source: 2016 AIME I Problem 10)*

5. Anh read a book. On the first day she read $n$ pages in $t$ minutes, where $n$ and $t$ are positive integers. On the second day Anh read $n + 1$ pages in $t + 1$ minutes. Each day thereafter Anh read one more page than she read on the previous day, and it took her one more minute than on the previous day until she completely read the $374$ page book. It took her a total of $319$ minutes to read the book. Find $n + t$. *(Source: 2016 AIME I Problem 5)*